Rules For The Collaboration Round

1. The use of all calculators, handheld and online is prohibited.
2. Pictures are not necessarily drawn to scale.
3. Discussion about problems from this test with other teams before submission and before the test submission deadline is strictly prohibited.
4. There are 15 questions on this test with a time constraint of 30 minutes.
5. Questions are not necessarily ordered by difficulty.
6. You team will be awarded 5 points for a correct answer, 0 points for a blank answer and 0 points for an incorrect answer.

Good Luck!

Thanks to our sponsors!

IMTC Collaboration Round

The IMTC Team

Problem C1

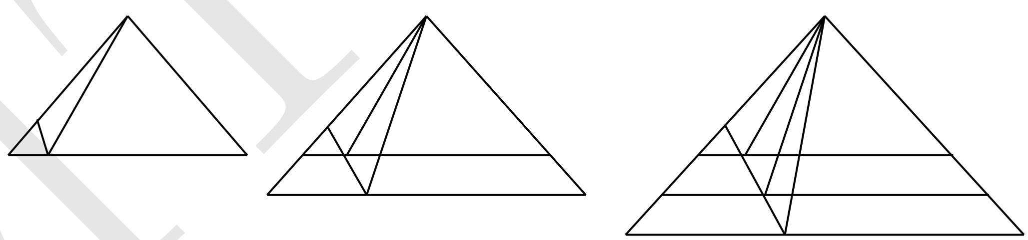
How many even 4 digit numbers have all distinct digits?

Problem C2

A child is playing with his calculator. He enters an integer number, then presses the square root button, $\boxed{\sqrt{\ }}$, to find the square root of the number. Then he floors the number by using the $\boxed{\lfloor x \rfloor}$ button to the largest integer that is smaller than it. He then takes this number as the input and repeats the process. After the third cycle, the output shows him the number 1 for the first time since he started playing the game. Let the positive difference between the largest and smallest number that the child put in the start be n. Find the value of n.

Problem C3

The first three stages of a developing model are shown. How many triangles, of any size, are there in the 20th stage?



Problem C4

Each edge of a cube is assigned to one of its adjacent vertices. Find the number of ways such that each vertex has at least 1 edge assigned to it.

Problem C5

The following is the chat between 2 people who met at a movie theatre:

A: I am 6. I have 3 cousins whose ages are all whole numbers. I can tell you that the product of their ages is 72 and the sum of their ages is the sum of our ticket numbers.
B: I know the sum of our tickets but I still do not know their ages.
A: 2 of my younger cousins have the same age.
B: Now I know their ages.

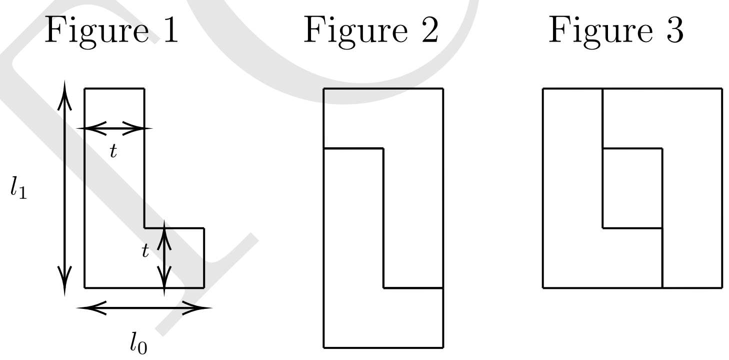
Find the sum of the ages of the 3 cousins of Person A.

Problem C6

Points A and B lie on the parabola $y = x^2$. Given that the slope of line AB is 50, find the x coordinate of the midpoint of line AB if it has length 50.

Problem C7

Two "L" shaped tiles are made as shown in Figure 1, with lengths $l_1 > l_0$ and thickness t. If the tiles are placed in such a way that they form a rectangle with no gap in between, they have an area of 66, as shown in Figure 2. If the tiles are shifted in such a way that they create a rectangle, but with a gap in the center, the area the two tiles and the gap take up is 72, as shown in Figure 3. What is $l_0 + l_1 + t$ if they are all integers?



Problem C8

Both $\overline{IM}$ and $\overline{TC}$ are 2 digit numbers where different letters represent different digits. If $\overline{IM} + \overline{TC} = 99$, find the sum of all positive distinct units digits of $(I + M + T + C)^{I \cdot M \cdot T \cdot C}$.

Problem C9

There exist two geometric series, $a_1, a_2, a_3, \cdots$ and $b_1, b_2, b_3, \cdots$. If the following sums are true, find $\frac{b_1}{a_1} + \frac{b_2}{a_2} + \frac{b_3}{a_3} + \cdots$.

- $a_1 + a_2 + a_3 + \cdots = 3$
- $b_1 + b_2 + b_3 + \cdots = 4$
- $a_1b_1 + a_2b_2 + a_3b_3 + \cdots = \frac{4}{3}$
- $\frac{a_1}{a_2} + \frac{b_1}{b_2} = \frac{38}{15}$

Problem C10

Two isosceles triangles are connected by their identical bases of length 5. If the sum of their heights is 8, what is the maximum number of non-overlapping circles of radius 3 that can fit in the kite formed by the sides of the triangles other than the base?

Problem C11

A sphere O has diameter DC which has length 2. A and B are 2 points on the surface of this sphere such that line segment AB has length $\sqrt{2}$. It is known that $\angle BDC = \angle ADC = 45°$. Given that the volume of triangular pyramid $ABCD$ can be expressed as $\dfrac{p}{q}$ where $\gcd(p, q) = 1$, then find $p + q$.

Problem C12

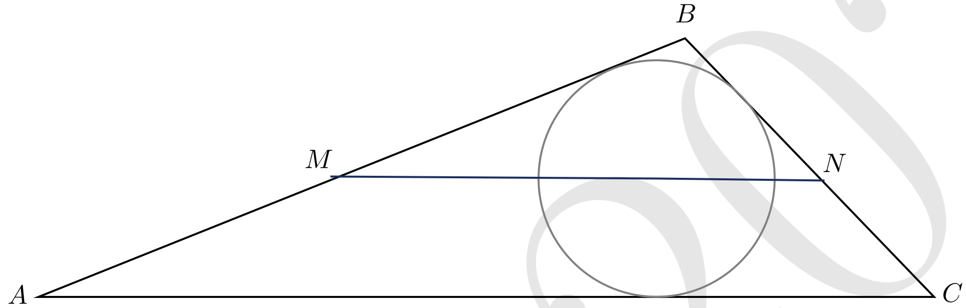
If $\frac{\log_a b}{9}$, a, and $4 \log_b a$ form an arithmetic series, find the minimum value of a. If the answer can be represented as the common fraction $\frac{x}{y}$, find $x + y$.

Problem C13

Circles with centers O and Q are tangent at M. Distinct lines AB and CD are tangent to both of the circles such that they do not pass through point M. Point N lies inside circle O such that $\angle ONQ = 90°$ and is closer to line AB than line CD. If it is known that $\angle AOC = 120°$ and the area of circle Q is 25π, then let the area of the region that lies in quadrilateral $QOAB$ but is outside both of the circles be $w\sqrt{x} - \dfrac{y}{z}\pi$ in simplest form where x reaches its minimum and $\gcd(y, z) = 1$. Given that w, x, y and z are integers, find $wxyz$.

Problem C14

The incircle O of triangle ABC has radius 2. Let M and N be points on AB and BC so that they are the intersections of the line that passes through the center of O and $MN \parallel AC$. If $MN = 7$ and $AM = 4$, find the area of trapezoid $AMNC$. If the answer can be expressed as $a + b\sqrt{3} + c\sqrt{5}$, find $a + b + c$.



Problem C15

There exists 2 functions $f(x)$ and $g(x)$ which both have the domain $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, have a range which is a subset of the domain, and that for all integers 1 to 10, $f(x) > g(x)$. For integers $1 \leq i \leq 10$, let point F_i be the point $(i, f(i))$ on the coordinate plane and let point G_i be the point $(i, g(i))$. For all possible functions $f(x)$, and $g(x)$, find the sum of all distinct possible areas of polygon $F_1F_2F_3...F_9F_{10}G_{10}G_9G_8G_7G_6...G_3G_2G_1$.