IMTC Division B

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Daily Challenge with Po-Shen Loh

Wolfram Language™

BRILLIANT

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Problem 1

At the Math Aquarium, there are 70 more Mathaladons than Angler Fish. Given that the number of Mathaladons is 3 times the number of Angler Fish, how many Mathaladons are there?

Problem 2

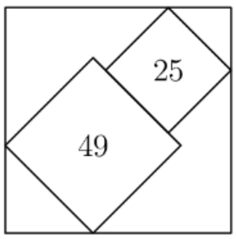
Jack is thinking of a positive integer with 3 distinct digits. Given that its units digit is a 5 and it can be expressed as the square of an integer, what number is Jack thinking of?

Problem 3

At IMTC, it is true that in X days, 4 people can write 24 problems, and in 17 days, Y people can write 54 problems. Given that $\frac{X}{Y}$ can be written in simplest form as $\frac{m}{n}$, find $m+n$.

Problem 4

Two squares with areas 49 and 25 are inscribed inside a larger square in such a way that the centers of the smaller squares lie on a diagonal of the larger square. Find the area of the larger square.



Problem 5

Bob has a deck of 6 cards labeled with a distinct integer from 1 to 6. Every second, he gives Alice a card, upon which Alice makes a statement about the sum of her cards. She says the following, in order:

The sum of my numbers is prime.
The sum of my numbers is a perfect square.
The sum of my numbers is a perfect cube.
The sum of my numbers is my age.
The sum of my numbers is a perfect square.
The sum of my numbers is 21.

Find the sum of the possible values of Alice's age.

Problem 6

A three-digit positive integer is chosen at random. Then the probability that the product of its digits will be prime can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Problem 7

Martha chooses a positive integer n for which the sum of the digits of n is the same as the product of the digits of n. She then adds 101 to her number, and finds that it still satisfies this property. Find n.

Problem 8

The numbers 1 to 9 are placed in a 3×3 grid so that any two squares that share an edge sum to an odd number. Moreover, the two main diagonals of this grid both sum to multiples of 3. In how many ways can we do this?

Problem 9

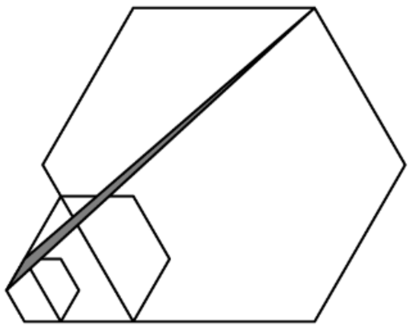
Geoff chooses a three digits a, b, c from 1 to 9, and writes the seven numbers

$$a, b, c, a+b, b+c, c+a, a+b+c.$$

He finds that exactly five of these numbers are perfect squares. Find the product of the other two.

Problem 10

Three hexagons are drawn as shown. The shaded triangle has area 21. Then the area of the smallest hexagon can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

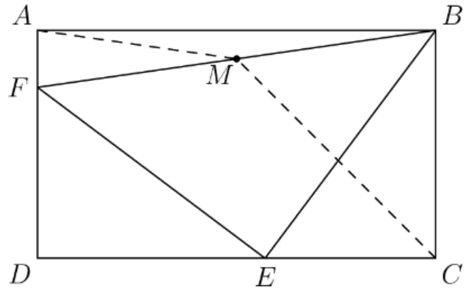


Problem 11

Let $P(x)$ be a quadratic polynomial such that the product of the roots of P is 20. Real numbers a and b satisfy $a+b=22$ and $P(a)+P(b)=P(22)$. Find a^2+b^2.

Problem 12

In rectangle $ABCD$, points E and F are chosen on sides CD and DA, respectively, such that triangle BEF is an isosceles right triangle with vertex E. Let M be the midpoint of BE. If $MA=5$ and $MC=7$, find the area of $ABCD$.



Problem 13

Let x and y be real numbers satisfying the system

$$2^x - 81y^2 = 0$$
$$3^x - 512y^3 = 0$$

Then y can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Problem 14

Let $\mathcal{S}$ be the set of the first 8 prime numbers. For each subset $\mathcal{T}$ of $\mathcal{S}$, let $f(\mathcal{T})$ be the remainder when the product of the elements of $\mathcal{T}$ is divided by 6. Find the sum of $f(\mathcal{T})$ over all subsets of $\mathcal{S}$. Note: for the empty set, we define $f(\emptyset) = 1$.

Problem 15

Let $ABCD$ be a rectangle and let ω be the circle with center A that passes through C. If line BD intersects ω at points P and Q satisfying $PB = 10$ and $DQ = 12$, compute the sum of all possible values of BD.