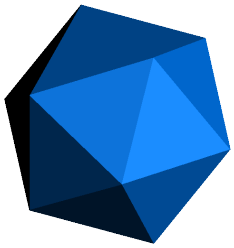


IMTC

Division B, Summer 2023

Thanks to our Sponsors!

Problem 1

In a room with 300 people with only Americans and Canadians, there are 200 Americans and 230 Canadians. Determine the number of non-American Canadians.

Problem 2

Four distinct integers from 1 to 9 are written on a piece of paper. Given that the first two numbers multiply to 24 and the third and fourth numbers multiply to 72, find the sum of the digits written.

Problem 3

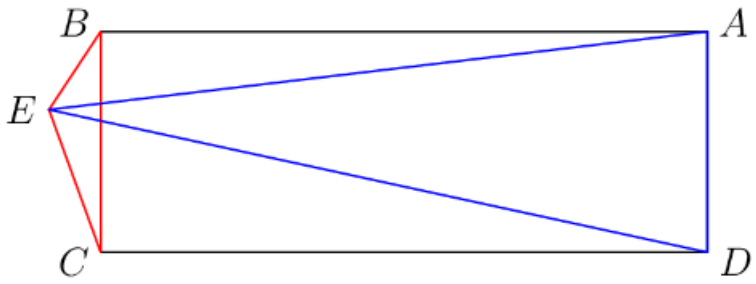
The second largest divisor of a number is 39, and its third smallest divisor is 3. What is the number?

Problem 4

Find the three digit number with units digit 3 which is the average of a power of 17 and a power of 23.

Problem 5

Point E is chosen to the left of line BC in rectangle $ABCD$ such that triangles $\triangle EBC$ and $\triangle EAD$ have areas 3 and 12, respectively. Find the area of rectangle $ABCD$.



Problem 6

Find the unique 3-digit multiple of 13 whose digits sum to 23.

Problem 7

Two regular hexagons are inscribed inside a rectangle as shown. The smaller hexagon has area 1. Find the area of the larger hexagon.

Problem 8

Let a, b, c, d be nonzero digits such that

$$\underline{abc} + \underline{acb} + \underline{abd} = \underline{ddd}$$

. Find the product of all possible values of $a + b + c + d$.

Problem 9

Given that there are exactly 15 square numbers between $36a$ and $49a$, inclusive, find the greatest possible value of a.

Problem 10

Find the number of pairs of nonnegative real numbers (a, b) that exist so that $a^2 + b^2 = 250000$ and $a + b$ is an integer.

Problem 11

Starting from $(0, 0)$, a bug wants to travel to $(10, 0)$ via lattice points. However, he cannot travel to coordinates that have an odd sum. Given that he can jump for a travel for a maximum distance of 1.5 units, determine the number of possible ways he can reach his destination using paths with the least possible distance.

Problem 12

Let $ABCD$ be a quadrilateral with $\angle A = 60, \angle C = 120$, $AC = 6$, and $BC = CD = 4$. Find the square of the area of triangle ABD.

Problem 13

How many 4 digit numbers exist so that the sum of the first two digits is divisible by 3, the sum of the second and third digits is divisible by 2 and the sum of the third and fourth digit is divisible by 5?

Problem 14

Trapezoid $ABCD$ has diagonals of length 6 and 8. It is given that the distance between the midpoints of AB and CD is the same as the distance between the midpoints of BC and DA. Find the area of $ABCD$.

Problem 15

Suppose there exist quadratic polynomials $P(x)$, $Q(x)$, and $P(x) + Q(x)$ with vertices of $(4,\ 5)$, $(9,\ 15)$, $(8,\ 80)$ respectively. Find $P(2) + Q(3)$.